# The 10% Rule for Transmission lines

When operated at a single frequency, the input impedance of a lossless transmission line of length $\ell$ and characteristic impedance $Z_o$ terminated by a load impedance of $Z_L$ is:

$$Z_{in}(\ell) = Z_0 \frac{Z_L + jZ_0 \tan[2\pi(\ell/\lambda]}{Z_0 + jZ_L \tan[2\pi(\ell/\lambda]}$$

where $\lambda = \dfrac{2\pi}{\beta}$ is the wavelength on the line at the operating frequency.

It is more or less accepted in RF circles that a transmission line between a source and a load becomes apparent when the line length exceeds 1/10$^{th}$ of the wavelength at the frequency of operation. This means that to ensure the highest frequency components of a time-domain signal will be unaffected requires that the line length be bounded by

$$\ell < \frac{1}{10}\lambda_{\min}$$

Where $\lambda_{\min}$ is the minimum wavelength, which occurs at the maximum appreciable frequency contained in the signal:, $\lambda_{\min} = \dfrac{u}{f_{\max}}$, where $u$ is the velocity of the line. Substituting, this yields:

$$\ell < \frac{1}{10}\frac{u}{f_{\max}}$$

where $f_{\max}$ is the maximum significant frequency of the signal. But noting that $\ell / u$ is simply the one-way time-delay $T$ of the line, $T$ must be bounded by:

$$T < \frac{1}{10}\frac{1}{f_{\max}}$$

Finally, it is known that the spectrum of trapezoidal pulse train with a rise time of $\tau_r$ falls at 40dB/decade beyond the break at frequency $f_b = \dfrac{1}{\pi\tau_r}$, so it is reasonable to assume that the vast majority of the signal energy is contained up to $f_{\max} = \pi f_b$ (approximately ½ a decade beyond $f_b$), which would mean:

$$T < \frac{1}{10}\frac{1}{f_{\max}} = \frac{\pi}{10}\tau_r$$